XXXIX.

On the Eclipse of the Sun of Sept. 17, 1811, *with the longitudes of several places in this country, deduced from all the observations of the Eclipses of the Sun and Transits of Mercury and Venus, that have been published in the Transactions of the Royal Societies of Paris and London, the Philosophical Society held at Philadelphia, and the American Academy of Arts and Sciences.*

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I OBSERVED the eclipse of Sept. 17, 1811, in the garden adjoining my house in Salem, about 300 feet S S W from the meeting house where the Rev. Doctor Barnard officiates. An excellent chronometer, made by Grimalde, was used in this observation. The regulation was made by equal altitudes of the sun, observed in the morning and evening, for several days before and after the eclipse, by an accurate sextant made by Ramsden, and a level surface formed by a bowl of Barbadoes tar. The observation of Sept. 16, 17, and 18, gave the following result.

	App. noon by chronometer.	Too slow for app. time.	Diff.
Sept. 16, by 28 observations	10*h*. 47′ 52″·0	1*h*. 12′ 08″·0	
17, " 48 "	10 47 22 ·9	1 12 37 ·1	29″·1
18, " 36 "	10 46 52 ·5	1 13 07 ·5	30 4

During the whole eclipse the weather was very clear, not a cloud was to be seen, and there was but little wind. The telescope used was a four feet achromatic, with a power of about 30. An assistant seated near the telescope counted the seconds from the chronometer, at the times of the beginning and end of the eclipse. About two minutes before the commencement of the eclipse, the part of the sun's

limb where the first contact took place was brought in the centre of the field of view, and kept there till the first impression on the limb was observed at 11*h.* 42′ 36″ per chronometer. The end of the eclipse was at 2*h.* 46′ 18″ per chronometer. The first contact appeared to be instantaneous; it seemed as if there could not have been an error of 1″ in this time. The end was not quite so well defined; the moon appeared to remain 2″ or 3″ on the sun's limb. Fahrenheit's thermometer in the shade stood at 71° at 1*h.* P. M; at 2*h.* 43′ the mercury had fallen four degrees, and at 4*h.* had again risen to 71°.

The chronometer at noon Sept. 17 was too slow for apparent time 1*h.* 12′ 37″·1, and the daily variation was 30″·4. A proportional part corresponding to the beginning of the eclipse is 1″·2, and at the end 5″; making the chronometer too slow at those times 1*h.* 12′ 38″·3 and 1*h.* 12′ 42″·1; which, added to the observed times, give

Beginning of the eclipse	0*h.* 55′ 14″·3	apparent time.
End of the eclipse	3 59 00 ·1	

The latitude of the place of observation is 42° 33′ 30″ N. The longitude 53 seconds in time East from Harvard Hall in Cambridge, as found by a trigonometrical survey, made by Professor Farrar from Cambridge to Boston Light-House, and by myself from the Light-House to Salem. This agrees with the calculation of Seth Pease, Esq. from his survey of the Post Roads, and accords very nearly with Holland's map.

The eclipse was also carefully observed at Salem, by Mr. Samuel Lambert, at a place, which by measurement was found to be 6″ N and 2″·7 in time E from the place where my observations were made. Hence the place of this observation is in the latitude of 42° 33′ 36″ N, and in the longitude of 55″·7 E from Cambridge. The chronometer used by Mr. Lambert was an excellent one, made by

Barraud, carefully regulated for several days before and after the eclipse, by equal altitudes of the sun, by a sextant and a surface of molasses. To verify the regulations of the time-keepers used in our observations, we compared them together for several days before the eclipse, and ascertained their relative rate of going. They were also compared a few hours before the eclipse and immediately after it, and we found, that at the beginning and end of the eclipse, the regulation deduced from his observations agreed exactly with mine. The telescope used by Mr. Lambert was a Gregorian Reflector, 18 inches in length, and his manner of observing and counting time was similar to that in my observations. The result of his observation is,

Beginning of the eclipse	$0h.\ 55'\ 24''\cdot3$	apparent time
End of the eclipse	3 59 01 ·1	

From these observations Mr. Lambert calculated the ecliptic conjunction to be at $2h.\ 13'\ 31''\cdot6$ apparent time, using the elements as given in the Nautical Almanac, without correcting for the errors in the moon's longitude and latitude. This differs but one second from the result found by using Burg's and Delambre's Tables, corrected for the errors in longitude and latitude, as will be shown hereafter.

The observations made at Salem are useful in finding the error of the lunar tables in longitude, and the apparent time of the conjunction under the meridian of Greenwich. To do this, it will be necessary to ascertain the longitude of the places of observation from Greenwich, to as great a degree of accuracy as possible; and as these longitudes are made to depend on that of Harvard Hall in Cambridge by means of the above survey, it will be necessary in the first place to calculate the longitude of that University, from the observations made at Cambridge, Chelsea, Salem, and Newbury.

On the longitude of Cambridge University.

The late Rev. President Willard, in volume i. page 60 of the Memoirs of the American Academy of Arts and Sciences, made the difference of meridians between Greenwich and Cambridge $4h.\ 44'\ 30''\frac{2}{3}$, by the mean of the observations of the solar eclipses in 1766 and 1778, and the transit of Mercury in 1743. In these calculations he supposed the difference of meridians between Greenwich and Paris to be $9'\ 16''$ in time instead of $9'\ 21''$, which is now used by astronomers.* This causes a difference of $5''$ in the longitude of Cambridge, deduced from the transit of 1743. The ratio of the polar to the equatorial diameter of the earth was assumed to be $\frac{229}{230}$, whereas the latest calculations of Burg and La Place, from the lunar equations arising from the oblate figure of the earth, make it $\frac{304}{305}$, or nearly $\frac{299}{300}$, as supposed by La Lande. For these reasons I concluded to recalculate these and other late observations, with the new tables of Burg, Delambre, and La Lande, as published in the third volume of Vince's Astronomy, always using the ratio of the diameters of the earth as assumed by La Lande, $\frac{299}{300}$.

The first observation calculated by President Willard is an eclipse of the sun of Aug. 5, 1766, at Greenwich, in the latitude of $51^\circ\ 28'\ 40''$, observed by Dr. Maskelyne and his assistant. The mean of

* La Lande, in the third edition of his Astronomy, supposed the difference of meridians of Greenwich and Paris to be $9'\ 18''{\cdot}8$, in conformity to the calculation of General Roy, from the survey made some years ago for connecting the two observatories. (Vide Phil. Trans. 1790, page 223.) A more accurate estimate from the same data by Mr. Dalby, made it $9'\ 20''{\cdot}4$. (Phil. Trans. 1791, Part 2, p. 245.) Delambre, in his Solar Tables, published in 1806, supposes the difference of meridians to be $9'\ 21''$.

their observations made the beginning at Greenwich 5*h.* 29′ 57″, and the end at 7*h.* 11′ 33″·5 apparent time. The beginning at Cambridge, in the latitude of 42° 23′ 28″, observed by Dr. Winthrop, was 11*h.* 39′ 23″, end 2*h.* 45′ 09″ apparent time. These make the conjunction at Greenwich at 5*h.* 43′ 57″·6, and at Cambridge at 0*h.* 59′ 27″·0 apparent time; whence the difference of meridians is 4*h.* 44′ 30″·6.

The second observation is an eclipse of the sun of June 24, 1778. The beginning at Greenwich, observed by Dr. Maskelyne, was 3*h.* 40′ 11″, and the end at 5*h.* 25′ 12″ apparent time. This was compared with the Rev. Mr. Paysons's observations at Chelsea, in the latitude of 42° 25′ 11″ N, and 26 seconds in time east from Cambridge: The beginning was at 9*h.* 6′ 42″, the end at 11*h.* 38′ 23″ apparent time. President Willard used only the time of the end of the eclipse at Chelsea, but upon examination it was found that the beginning gave very nearly the same result as the end, and it was thought proper to use both observations. The conjunction at Greenwich deduced from these observations is at 3*h.* 35′ 52″·9, and that at Chelsea at 10*h.* 51′ 50″·9 apparent time. Whence, by allowing 26″ for the difference of meridians of Cambridge and Chelsea, the difference of meridians of Greenwich and Cambridge by this eclipse will be obtained, 4*h.* 44′ 28″·0.

The third observation used by President Willard is the transit of Mercury of Nov. 5, 1743, observed at Cambridge by Professor Winthrop: second internal contact at 8*h.* 17′ 5″ A. M. second external contact at 8*h.* 18′ 58″ A. M. apparent time. The same was observed at Paris, in the latitude of 48° 50′ 14″ nearly, by Messrs. Maraldi, Cassini senior and junior, La Caille and Le Monnier, as in the Memoirs of the Royal Academy of Arts and Sciences of Paris for 1743,

The mean of all these observations, reduced to the meridian of the Observatory of Paris, give for the first internal contact 8*h.* 40′ 40″·2 A. M. second internal contact 1*h.* 10′ 15″·7, second external contract 1*h.* 12′ 10″·5 apparent time. These times differ a little from those given in volume 1, page 53 of the Memoirs of the American Academy of Arts and Sciences, owing to a difference in the reduction of the observations of the internal contacts, observed by Cassini senior, and the external contact by Le Monnier. From these observations, computing the parallaxes for each of the contacts, the difference of meridians between Paris and Cambridge is 4*h.* 53′ 53″·3,* and by subtracting the difference of meridians of Paris and Greenwich, 9′ 21″, there remains the difference of meridians of Greenwich and Cambridge, 4*h.* 44′ 32″·3.

The fourth observation is the solar eclipse of April 3, 1791. This was observed at Cambridge by Professor Webber. The beginning at 6*h.* 1′ 27″, formation of the annulus 7*h.* 8′ 7″, breaking of the annulus 7*h.* 12′ 56″, end of the eclipse 8*h.* 28′ 26″ A. M. apparent time. The observations at Greenwich were, the beginning 0*h.* 18′ 40″·0, the end 3*h.* 6′ 47″ apparent time. At the Royal Observatory at Paris, beginning 0*h.* 33′ 37″·4, end 3*h.* 17′ 36″·0 apparent time. The observations at Greenwich, make the conjunction at 0*h.* 42′ 1″·3 apparent time. Those at Paris 0*h.* 51′ 20″·1 apparent time, from which subtracting 9′ 21″, leaves the apparent time of the conjunction at Greenwich 0*h.* 41′ 59″·1. The mean of this and the former estimate is 0*h.* 42′ 00″·2, the difference between this and the time of the conjunction at Cambridge 19*h.* 57″ 29″·1, leaves the difference of Meridians of Cambridge and Greenwich 4*h.* 44′ 31″·1.

* In the computation in vol. i. page 55 of the Memoirs of the Academy, the longitude of the nonagesimal is too great by 30°, which renders the parallaxes there calculated incorrect. There is a similar mistake in vol. ii, p. 28.

The fifth observation is the total eclipse of the sun of June 16, 1806. The time of conjunction deduced from my observations at Salem, compared with the time of conjunction at Paris, computed by La Lande, gives, by allowing 53 seconds for the difference of meridians of Salem and Cambridge, the longitude of Cambridge 4*h.* 44′ 24″·9 W from Greenwich, as is shown in the additional observations on that eclipse given in this memoir.

The sixth observation is the transit of Venus of June 3, 1769. The observations at Newbury and Cambridge make the conjunction at Cambridge at 5*h.* 20′ 18″·6, and those at Greenwich and Paris make the conjunction at Greenwich at 10*h.* 4′ 49″·7 apparent time. The difference is the longitude of Cambridge by this observation 4*h.* 44′ 31″·1, as will be shewn hereafter.

Upon examination of the transactions of the Royal Societies of London and Paris, those of the Society held at Philadelphia and the Memoirs of the American Academy of Arts and Sciences, two more corresponding observations were found. The first was that of the transit of Mercury of Nov. 12, 1782, but this was very poorly adapted to determine with accuracy the difference of meridians. For the planet entered but 31 seconds on the sun's disc, and the situation was such that a small error in the latitude of the planet would cause a great error in the difference of meridians, and it entered so obliquely on the sun's limb that it was difficult to determine the precise moment of the contacts. This was particularly the case at Paris where the sun was low: the observers there differed above 4 minutes in the time of the first internal contact and above 2 minutes in the second external contact. In the transit of 1789, the planet was more favourably situated, as it respects its latitude, but the sun set before the end of the transit at Paris, and the weather prevented making an observation of the beginning of the transit; the first internal contact was however observ-

ed there by astronomers in an interval of clear weather. This observation being made under such unfavourable circumstances, and making the longitudes nearly 20 seconds less than the other observations, it was thought proper not to take it into the computation, and as no other observations were known, that could be used in this calculation, the mean of the preceding values was taken as the true longitude of Cambridge.

Solar Eclipse—	Aug. 5, 1766. (2 Obs. Green. 2 Obs. Camb.)	$4h. 44' 30'' 6$
	June 24, 1778. (2 Obs. Green. 2 Obs. Chelsea.)	28 0
	April 3, 1791. (2 Obs. Paris, 2 Obs. Green. 4 Obs. Camb.)	31 1
	June 16, 1806. (Several in Europe, 4 Obs. in Salem.)	24 9
Transit of Mercury—	Nov. 5, 1743. (4 Obs. Paris, 2 Obs. Camb.)	32 3
Transit of Venus—	June 3, 1769. (2 Green. 1 Paris, 1 Camb. 1 Newbury.)	31 1
	Mean of above thirty observations	4 44 29 7

As the difference of latitude of Cambridge and Salem was found by a trigonometrical survey to be less than by astronomical observations, it was thought proper to estimate the effect of a supposed error in the latitude of Salem, Cambridge or Chelsea. To do this the preceding calculations were repeated with latitudes increased by one minute, and it was found that the longitudes, deduced from the observations of 1806, 1743 and 1769, were not varied a tenth of a second by this change, and the corrections of the observations of 1766, 1778, and 1791, were respectively $-1''{\cdot}5 + 1''{\cdot}2 + 1''{\cdot}8$, therefore the mean longitude just found would be increased $\frac{1}{4}$ of a second by an increase of 1 minute in the latitude of Cambridge, consequently the error in the longitude, arising from this source, must be wholly insensible, since there cannot be an error of a minute in the latitude of that place.

Hence the difference of meridians between Harvard Hall in Cambridge and the Royal Observatory of Greenwich may be assumed as equal to $4h.\ 44'\ 29''{\cdot}7$, and from the near agreement of these observa-

tions I am inclined to believe that *this longitude is more accurately ascertained than that of any other place in the United States.* The calculation of President Willard differs about a second from the above estimate. Mr. Ferrer in vol. vi. page 359 of the Transactions of the American Philosophical Society, computed the longitude by combining the observations of the solar eclipse of 1791, the transit of Venus of 1769, and the transits of Mercury of 1782 and 1789, making the longitude greater by $2''\cdot3$, but this difference is less than was to be expected in using an observation so liable to error as that of the transit of Mercury of 1782.

As several emersions of the first and second satellites of Jupiter have been observed at Cambridge by Professors Winthrop and Williams, and published in the Philosophical Transactions and in the Memoirs of the American Academy, my curiosity was excited to know how near the longitude could be ascertained by comparing those observations with the times computed from Delambre's tables of the satellites, published in the third edition of La Lande's Astronomy. The result of this calculation is in the following table :—

	App. time.				Longitude.		
	d.	h.	′	″	h.	′	″
Emersion 1st satellite, by Dr. Winthrop, 1768 April	25	9	13	52	4	43	33·4
May	18	9	27	27	4	44	25·3
June	10	9	37	25	4	45	02·8
July	3	9	45	54	4	44	41·4
1769 May	14	10	19	07	4	43	09·7
Aug.	23	7	31	50	4	43	58·1
Dr. Williams, 1782 July	3	12	09	53	4	44	21·0
Aug.	27	9	6	25	4	44	38·4
Sept.	12	7	31	29	4	46	43·0
Emersion 2d Satellite, by Dr. Winthrop, 1769 June	7	9	01	15	4	44	40·1
Dr. Williams, 1782 June	25	9	48	30	4	44	26·6
July	2	12	21	54	4	44	11·2
Aug.	28	9	03	49	4	44	27·9
Mean					4	44	29·1

The mean of these thirteen observations makes the difference of meridians of Greenwich and Cambridge 4*h.* 44′ 29″·1, varying but ⅗ of a second from the preceding calculation. The near agreement of these results may however be considered as wholly accidental, since the number of observations is small, and the differences from each other above three minutes. Indeed no great accuracy is to be expected by this method, unless the number of observations be very great, and the number of emersions and immersions nearly equal. In proof of this, I shall give the longitudes deduced from the emersions of the first, second, and third satellites, observed at Chelsea by the Rev. Mr. Payson, and publised in the first volume of the Memoirs of the American Academy of Arts and Sciences. These observations being reduced to the meridian of Cambridge by allowing 26 seconds for the difference of meridians, give the longitude of Cambridge as in the following table.

	App. time.	Longitude.
	d. h. ′ ″	
1st Satellite	1779, April 22 10 36 37	4 46 07·8
	1779, May 8 8 56 53	4 45 40·1
	1779, May 15 10 51 41	4 43 39·6
	1779, June 23 9 16 14	4 43 40·9
2d Satellite	1779, May 29 8 57 34	4 43 54·6
	1779, June 30 8 38 49	4 42 50·2
3d Satellite	1779, May 16 8 53 54	4 41 42·1
	1779, May 23 12 52 14	4 41 57·4
	1779, June 28 8 35 35	4 42 44·8
	Mean	4 43 35·3

The mean of these nine observations differs nearly 54 seconds from that of the former set. Hence we perceive the uncertainty of this kind of observations. These calculations were not examined with much care, as it was found that the results were not sufficiently exact to be used in the present computation.

The longitude of Harvard Hall in Cambridge being 4*h.* 44′ 29″·7

West from Greenwich, that part of Salem where my observations of the eclipse of Sept. 17, 1811, were made is in the longitude of *4h.* 43′ 36″·7, and where Mr. Lambert observed must be in the longitude of *4h.* 43′ 34″·0. These longitudes are used in reducing the Salem observations.

Observations at Salem, Sept. 17, 1811.

The following elements of the eclipse of Sept. 17, 1811, corresponding to the beginning and end of the eclipse as observed by me in Salem, were calculated from the tables of Burg and Delambre, published in Vince's Astronomy.

	Beginning.	End.
	h. ′ ″	*h.* ′ ″
Apparent times of observation - -	0 55 14·4	3 59 00·1
Mean times of observation - -	0 49 48·4	3 53 31·5
Longitude west from Greenwich - -	4 43 36·7	4 43 36·7
Mean times of observation reduced to Greenwich	5 33 25·1	8 37 08·2
☉'s longitude from the apparent equinox -	173°53′21″·4	174°00 50·4
☉'s horizontal parallax - - -	8·77	8·77
☉'s semidiameter — Irradiation 3″·5 -	15 53·75	15 53·78
☉'s horary motion - - -	2 26·55	2 26·56
Apparent obliquity of the ecliptic - -	23 47 41·9	
☽'s longitude from the apparent equinox -	173 18 13·1	174 48 41·5
☽'s latitude north increasing - -	33 18·0	41 37·5
☽'s horizontal parallax — 4″·92 reduction for Salem	53 54·98	53 55·88
☽'s Semidiam. — inflex. 2″+aug. [10″·53 & 5″·70]	14 52·80	14 48·22
☽'s horary motion in longitude* - - -	29 32·41	29 33·43
☽'s horary motion in latitude - -	2 43·33	2 42·91
☽'s horary motion from the sun in longitude	27 05·86	27 06·87
☽'s parallax — ☉'s parallax - - -	*h.* 53 46·21	*h.* 53 47·11
☉'s right ascension - - -	11 37 33·9	11 38 01·3
Horary increment of ☉'s right ascension in time	8·95	
Horary increment of ☽'s horary motion in long.	0·32	0·36
Horary decrement of ☽'s horary motion in lat.	0·14	0·18
Horary increment of ☽'s horizontal parallax -	0·29	0·29
Horary increment of ☽'s semidiameter -	0·08	0 08
(☽—☉) parallax in longitude - -	4 37·9	17 18·3
(☽—☉) parallax in latitude - -	35 31·4	46 07·1

* These horary motions are for one hour *mean* time, all the calculations in this paper being made for mean time, the computed conjunction being reduced to apparent time at the end of the computation.

In calculating the time of conjunction in this and in the following observations, the moon's tabular longitude and latitude were first used without correction, and the time was found by calculation both for the beginning and end, the mean being taken for the time of conjunction. By repeating the process with those observations best adapted to the purpose it was found, that if the sun's longitude as given by Delambre's tables was correct, it would be necessary to decrease the moon's longitude as given by Burg 12″·0, and to decrease the tabular latitude by 10″·6. The error in longitude was deduced from the observations at Salem, and in latitude from those of Nantucket, Monticello and Washington. Two micrometrical observations made at Nantucket near the middle of the eclipse, gave for the correction of latitude— 9″·2 and — 12″·4, mean — 10″·8 ; the observations of the internal contacts of the eclipse at Monticello gave — 10″·4, and those at Washington — 10″·6. The mean of all these observations makes the error of latitude — 10″·6. It may be observed that this correction makes but very little change in the calculated longitudes of the places of observation, because the times of conjunction is affected in nearly the same way at most of the places. Thus, it was found that the correction of the time of conjunction at Salem, Brunswick, Portland, and Nantucket was 0″·8 ; at Burlington and Rutland+0″·9 ; at New Haven+1″·0 ; at New Brunswick + 1″·1 ; at New York + 1″·3 ; at Washington + 1″·4 ; at Monticello+1″·9, and at Williamsburg (where the first internal contact was not observed) + 0″·3. Hence the corrections of longitude arising from this source are respectively 0″·0, 0″·1, 0″·2, 0″·3, 0″·5, 0″·6, 1″·1, and — 0″·5 ; most of which are so small as to be hardly worthy of notice.

The error of the moon's longitude — 12″·0 has no sensible effect on the calculated longitudes from Greenwich, the time of conjunction being decreased about a quarter of a second at all the places of observation mentioned in this memoir.

The moon's longitude and latitude being corrected, the conjunction at Salem from my observations was found to be at 2*h.* 13′ 28″·9* apparent time; this, added to the longitude from Greenwich, 4*h.* 43′ 36″·7, gives the conjunction at that place 6*h.* 57′ 05″·6. By Mr. Lambert's observation calculated in the same way the conjunction was at 2*h.* 13′ 32″·7, which added to the longitude 4*h.* 43′ 34″·0 gives the conjunction at Greenwich 6*h.* 57′ 6″·7. The mean of the two observations is 6*h.* 57′ 06″·1,* which is used in the rest of this memoir as the apparent time of the conjunction at Greenwich, not having any European observations by which that time could be obtained.

At the time of the conjunction at Greenwich 6*h.* 47′ 06″·1, the sun's longitude by the above elements from Delambre's tables was 173° 56′ 32″·4, the moon's longitude from Burg 173° 56′ 44″·4, which differ 12″·0, agreeing with what was assumed above. At the same time the moon's latitude by Burg was 36′ 50″·8, which decreased by 10″·6 gives the moon's true latitude at the time of conjunction 36′ 40″·2 N. These corrected values were used in calculating the following observations.

Observations at Nantucket by Walter Folger jun. Esquire, Sept. 17, 1811.

The place of observation was a little to the westward of the centre of the town of Nantucket in the latitude of 41° 15′ 32″ reduced 41° 04′ 10″ N.

	App. time.	(☽−☉)Par.in lon.	Par. in lat.	☽ S. D.Aug.
Beginning	1*h.* 02′ 04″·8	3′ 00″·5	35′ 12″·5	14′ 52″·92
End	4 05 20 ·6	−18 48 ·8	45 48 ·2	14 48 ·06

Hence the apparent time of conjunction at Nantucket was 2*h.* 16′ 34″·5, which subtracted from the time of conjunction at Greenwich

* A decrease of 1′ in the estimated latitude of Salem, would decrease this time 1″·4.

6*h*. 57′ 06″·1 leaves the longitude of Nantucket 4*h*. 40′ 31″·6 = 70° 7′ 54″ West from Greenwich.

Mr. Folger made several observations of the width of the lucid part of the sun during the eclipse. I selected two observations when the distance was the shortest, subject to the least change, most easy to measure and best adapted to ascertain the error of the moon's tabular latitude. These observations were at 2*h*. 30′ 50″ and 2*h*. 35′ 01″ mean time. The measured distances were 4′ 58″·9; 4′ 56″·6. The moon's augmented semidiameter 14′ 51″·03; 14′ 50″·92. The sun's semidiameter 15′ 53″·77. Hence the apparent distances of the centres of the sun and moon (= measured lucid part + ☽'s semidiameter—☉'s semidiameter) were 3′ 56″·16 and 3′ 53″·75. The parallaxes in longitude (☽—☉) were—10′ 00″·6 and—10′ 30″·7; in latitude 41′ 21″·9 and 41′ 36″·5. From these were deduced the apparent latitudes; and, by applying the parallaxes in latitude, the true latitudes 37′ 35″·2 and 37′ 43″·4 were obtained, which subtracted from the corresponding tabular latitudes 37′ 44″·4 and 37′ 55″·8 give the error of the tables in latitude, by the first observation —9″·2; by the second —12″·4; the mean of both is —10″·8.

At Monticello, Virginia, by the Hon. Thomas Jefferson, late President of the United States and President of the American Philosophical Society, Sept. 17, 1811.

The two internal contacts are useful in finding the error of the moon's latitude. These observations are as follow, the latitude being 38° 8′ N, reduced 37° 56′ 52″.

	App. time.	(☽-☉) Par. long.	Par. in lat.	☽ Aug. S. D.
Annulus formed	1*h*. 53′ 00″	—5′ 58″·2	36′ 51″·4	14′ 52″·55
Annulus broken	1 59 25	—6 53 ·5	37 17 ·6	14 52 ·40

The calculation was first made from these observations, without

correcting the moon's tabular latitude, and the apparent times of the conjunction deduced differed nearly a quarter of a minute. By repeating the calculation it was found that they would both give the same result 1*h.* 41′ 55″·7, by decreasing the latitude 10″·4. The error of Burg's tables in the latitude of the moon is therefore by this observation — 10″·4. By subtracting the time of conjunction 1*h.* 41′ 55″·7 from 6*h.* 57′ 06″·1, the longitude of Monticello is obtained 5*h.* 15′ 10″·4 = 78° 47′ 36″ W from Greenwich.

At Washington, Columbia, by Seth Pease, Esq. Sept. 17, 1811.

These observations were made in the latitude 38° 54′ N, reduced 38° 42′ 47″, and 1′ 27″ in space, or 5″·8 in time, W from the Capitol.

	App. times.	(☽−☉) Par. long.	Par. lat.	☽ Aug. S. D.
Beginning	0*h.* 22′ 08″·9	8′ 30″·7	30′ 48″·5	14′ 53″·62
Annulus formed	2 02 06 ·0	—6 51 ·9	37 55 ·3	14 52 ·25
Annulus broken	2 06 53 ·1	—7 31 ·8	38 14 ·3	14 52 ·13
End of the eclipse	3 36 52 ·8	–18 07 ·0	43 26 ·3	14 49 ·38

The apparent times of the conjunction deduced from the internal contacts, without correcting for the error of the moon's latitude, were 1*h.* 48′ 55″·1 and 1*h.* 48′ 19″·8, differing 35″·3. This difference is wholly corrected by decreasing the moon's tabular latitude 10″·6, which makes the time of conjunction by both observations 1*h.* 48′ 39″·6. The external contacts give 1*h.* 48′ 58″·2. Mean 1*h.* 48′ 48″·9, which, subtracted from 6*h.* 57′ 6″·1, gives the longitude of the place of observation 5*h.* 8′ 17″·2, from which subtracting 5″·8 there remains the longitude of the Capitol 5*h.* 8′ 11″·4, by this observation.

The eclipse of April 3, 1791, observed at Georgetown, by Mr. Ellicott, makes the longitude of the Capitol 5*h.* 8′ 4″·5, as will be shewn hereafter. The mean of both may be assumed as the longitude of the Capitol 5*h.* 8′ 8″·0 = 77° 2′ 00″ W from Greenwich.

An occultation of α Tauri, observed by Mr. Ellicott January 21, 1793, and one of *n* Pleiadum by Messrs. Bradley and Pease, October 20, 1804, might be used in calculating the longitude of this city, if corresponding observations under a known meridian could be obtained. To compare these times with those computed for Greenwich in the Nautical Almanac, would render the result liable to an error which might amount to nearly half a degree of longitude.

At William and Mary College, Williamsburg, by Professor Blackburn, Sept. 17, 1811.

Latitude of the place of observation 37° 15′ 20″ reduced 37° 4′ 17″ North.

	App. time.	(☽–☉) Par. long.	Par. lat.	☽ S. D. Aug.
Beginning	0*h*. 26′ 56″·20	7′ 06″·5	30′ 02″·4	14′ 53″·83
Annulus broken	2 13 12 ·29	—9 23 ·0	37 42 ·3	14 52 ·18
End of the eclipse	3 41 57 ·20	—19 48 ·4	42 51 ·1	14 49 ·32

Hence the apparent time of conjunction, by the mean of these three observations, was 1*h*. 50′ 17″·7, which subtracted from 6*h*. 57′ 6″·1 gives the longitude of William and Mary College 5*h*. 6′ 48″·4 = 76° 42′ 06″ W from Greenwich. The calculation of the longitude of this place from the transit of Mercury of Nov. 5, 1789, will be given in this memoir.

Near New Brunswick, New Jersey, by Mr. John Garnett, Sept. 17, 1811.

Mr. Garnett's account of the observation is as follows. "The "beginning was perceptible at 0*h*. 36′ 5″ apparent time, allowing 3″ "before it became sensible for the contact, it must have been at 0*h*. "36′ 2″. The end at 3*h*. 45′ 58″ apparent time. Latitude of the "place of observation 40° 30′ 0″ N, being 26″ N and 2″·5 in time W "from Columbia College." As this allowance of 3″ was not made

on the other observations of this eclipse, it was thought that the computed longitude would be more accurate if the uncorrected values were used, as in the following calculation.

	App. time.	(☽ – ☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	0*h.* 36′ 05″	6′ 48″·8	32′ 53″·1	14′ 53″·29
End	3 45 58	—17 46 ·9	44 38 ·2	14 48 ·89

Hence the apparent time of conjunction was at 1*h.* 59′ 09″·5, which subtracted from 6*h.* 57′ 6″·1 leaves the longitude of the place of observation 4*h.* 57′ 56″·6 = 74° 29′ 9″ W. Hence Queen's College, which is 38″ E from Mr. Garnett's, is in 74° 28′ 31″ W from Greenwich. This will be combined hereafter with the observation at New York.

At Yale College, New Haven, by Professor Day, Sept. 17, 1811.

The latitude of the place of observation is 41° 17′ 58″, reduced 41° 06′ 35″ N.

	App. time.	(☽ – ☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	0*h.* 44′ 12″·7	5′ 50″·8	34′ 04″·9	14′ 53″·10
End	3 51 56 ·7	—17 45 ·1	45 24 ·5	14 48 ·59

Hence the apparent time of conjunction was at 2*h.* 5′ 15″·0, and the longitude of Yale College 4*h.* 51′ 51″·1 = 72° 57′ 46″ W from Greenwich.

At Rutland, Vermont, by Doctor Williams, Sept. 17, 1811.

These observations were published in the Washingtonian, a paper printed at Windsor, Vermont. " The beginning of this eclipse was " at 12*h.* 41′ 46″, the end was at 3*h.* 46′ 31″. Little or no uncertainty " seemed to attend these observations. The clock was carefully reg- " ulated by corresponding attitudes of the sun, taken for several days " before, and on the day of the eclipse, corrected by the proper equation " of equal altitudes—The telescope a reflector with a magnifying pow-

" er of 53. The latitude of Rutland nearly 43° 36′ N." Reduced latitude 43° 24′ 32″.

	(☽—☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	7′ 07″·6	35′ 13″·8	14′ 52″·75
End	—15 28 ·5	46 02 ·8	14 48 ·60

Hence the apparent time of the conjunction at Rutland was 2*h.* 5′ 18″·5, which subtracted from 6*h.* 57′ 06″·1 leaves the longitude of Rutland 4*h.* 51′ 47″·6 W from Greenwich.

In the remarks on the eclipse of June 16, 1806, in this paper, it is shewn that the longitude of Rutland, deduced from that eclipse, is 4*h.* 51′ 52″·0. The mean of the observations of 1806 and 1811 give for the longitude of Rutland 4*h.* 51′ 49″·8 = 72° 57′ 27″ W from Greenwich.

Observations at Burlington, Vermont, by Professor Dean and John Johnson, Esq. Sept. 17, 1811.

The place of observation was the University of Vermont, in the latitude of 44° 28′ N, reduced 44° 16′ 32″. The beginning of the eclipse by the observations of Professor Dean was at 0*h.* 38′ 51″, by Mr. Johnson 0*h.* 38′ 39″. The end by Professor Dean 3*h.* 43′ 11″, by Mr. Johnson 3*h.* 43′ 25″. The mean of these observations is 0*h.* 38′ 45″ and 3*h.* 43′ 18″ mean time, which were used in calculating the time of conjunction.

	(☽—☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	7*h.* 55″·3	35′ 33″·0	14′ 52″·65
End	—14 32 ·1	46 16 ·4	14 48 ·63

Hence the apparent time of conjunction was 2*h.* 4′ 7″·8, which subtracted from 6*h.* 57′ 6″·1 leaves the longitude of the university of Vermont 4*h.* 52′ 58″·3=·73° 14′ 34″ W from Greenwich.

Observations at Portland, by the Rev. Mr. Nichols and others, Sept. 17, 1811.

The beginning of the eclipse at Observatory Hill was 0*h.* 57′ 32″ apparent time, and at Mr. Nichols' observatory one mile W from the former place 0*h.* 57′ 34″ mean 0*h.* 57′ 33″. The end at those places was at 3*h.* 58′ 48″·7 and 3*h.* 59′ 00″ mean 3*h.* 58′ 54″·3. The latitude of the place of observation nearly 43° 39′, reduced 43° 27′ 32″.

	App. time.	☽—☉ Par. long.	Par. lat.	☽ aug. S. D.
Beginning	0*h.* 57′ 33″·0	4′ 45″·1	36′ 20″·0	14′ 52″·59
End	3 58 54 ·3	—16 26 ·7	46 34 ·7	14 48 ·12

Hence the apparent time of conjunction was at 2*h.* 15′ 41″·5, which subtracted from 6*h.* 57′ 6″·1 leaves the longitude of Portland 4*h.* 41′ 24″·6, which makes Observatory Hill in the longitude of 4*h.* 41′ 22″ W from Greenwich.

Observations at Bowdoin College, Brunswick, Sept. 17, 1811, *by Professor Cleaveland.*

Latitude of the place of Observation 43° 53′ N, reduced 43° 41′ 32″.

	Mean time.	(☽—☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	0*h.* 53′ 52″·2	4′ 35″·4	36′ 35″·3	14′ 52″·54
End	3 55 20 ·2	—16 24 ·4	46 45 ·0	14 48 ·04

Hence the apparent time of conjunction was at 2*h.* 17′ 23″·1, which subtracted from 6*h.* 57′ 6″·1 leaves the longitude of Bowdoin College 4*h.* 39′ 43″·0 W from Greenwich. It will be seen in the addition to this memoir, that the end of the eclipse of the sun of June 16, 1806, made the longitude of this college 4*h.* 39′ 37″·3. The mean of these observations makes the longitude of that place 4*h.* 39′ 40″·1=69° 55′ 1″ W from Greenwich.

At New York, Columbia College, by Dr. Kemp, Sept. 17, 1811.

The latitude of the place of observation 40° 42′ 45″ N, reduced 40° 31′ 24″.

	App. time.	(☽ – ☉) Par. long.	Par. lat.	☽ aug. S. D.
Beginning	0*h.* 38′ 53″	6′ 26″·7	33′ 13″·3	14′ 53″·23
End	3 47 49	—17 33 ·1	45 04 ·6	14 48 ·71

Hence the apparent time of conjunction was at 2*h.* 1′ 22″·9 apparent time, which subtracted from 6*h.* 57′ 6″·1 gives the longitude of Columbia College New York 4*h.* 55′ 43″·2 by this observation. Mr. Garnett's observation at New Brunswick made the longitude 4*h.* 57′ 56″·6, from which subtracting the difference of meridians between that place and Columbia College 1′ 51″·3 in time (as computed from the survey of the Post Roads by Seth Pease Esq. corrected at the extreme points by Mr. Garnett) gives the longitude of Columbia College by observation 4*h.* 56′ 5″·3.

Mr. Ferrer, by a chronometer, found the longitude of Kinderhook, where the solar eclipse of 1806 was observed, 51″·3 E from his house in Partition Street, New York, and Albany 58″ E from the same house. These correspond to 51″·0 and 57″·7 in time E from Columbia College, and by adding these respectively to the longitudes of Kinderhook and Albany computed in this paper from the eclipse of 1806, viz. 4*h.* 55′ 9″·6 and 4*h.* 54′ 59″·3, give the longitude of Columbia College by these observations 4*h.* 56′ 0″·6 and 4*h.* 55′ 57″·0.

By the solar eclipse of June 26, 1805, computed in this memoir, the longitude of Columbia College is 4*h.* 56′ 14″·8.

The mean of these five observations gives the longitude of Columbia College 4*h.* 56′ 00″·2 W from Greenwich, being 2″·8 less than the estimate of Mr. Ferrer in vol. vi. p. 360, of the Transactions of the American Philosophical Society of Philadelphia.

Additional observations on the total eclipse of June 16, 1806.

The mean times of this observation at Salem, given in the 20th page of the 3d volume of the Memoirs of the American Academy of Arts and Sciences, were estimated by using the equation of time given in the Nautical Almanac for 1806, which I have since found exceeds that in Delambre's new solar tables by 1″·2. The equation at the time of the conjunction by the Nautical Almanac is 6″·7 and by Delambre's tables 5″·5. The apparent times of these observations at Salem are, the beginning 10*h*. 6′ 18″·1, beginning of total darkness 11*h*. 25′ 19″·4, end of total darkness 11*h*. 30′ 7″·3, end of the eclipse 0*h*. 50′ 34″·6. The place of observation is 53 seconds in time east from Cambridge, as found by the survey made by Professor Farrar and myself. The elements at the beginning and end of the eclipse by the tables of Delambre and Burg are as follows.

	Beginning.	End.
	h. ′ ″	*h*. ′ ″
Apparent times of observation at Salem -	22 06 18·1	0 50 34·6
Mean times of observation - -	22 06 22·8	0 50 40·8
Longitude west from Greenwich - -	4 43 36·7	4 43 36·7
Mean times of observation reduced to Greenwich	2 49 59·5	5 34 17·5
☉'s right ascension - - -	5 36 49·8	5 37 18·3
☉'s longitude from the apparent equinox -	84°41 01·7	84 47 33·9
☉'s horizontal parallax - - -	8·66	8·66
☉'s semidiameter — Irradiation 3″·5 -	15 42·54	15 42·53
☉'s horary motion - - -	2 23·15	2 23·15
Apparent obliquity of the ecliptic - -		23 27 53·0
☽'s longitude from the apparent equinox -	83 49 33·2	83 30 02·2
☽'s latitude north decreasing - -	24 24·9	15 08·1
☽'s horizontal parallax — reduction for Salem 5″·49	60 08·71	60 11·81
☽'s Semidiam — inflex. 2″+aug. [15″·22 & 16″·37]	16 39·66	16 41·64
☽'s horary motion in longitude - - -	36 39·69	36 43·43
☽'s horary motion in latitude - -	3 22·24	3 22·98
☽'s horary motion from the sun in longitude	34 16·54	34 20·28
☽'s parallax — ☉'s parallax - - -	60 00·05	60 03·15

Hence the conjunction at Salem by the mean of the four observations made there was at 11*h.* 37′ 13″·1 A. M. apparent time, which differs but a fraction of a second from that found in page 28 of my former memoir; and as all the American observations would be affected in nearly the same way, it was thought unnecessary to recalculate them on account of the small variations arising from the use of these new tables. Subtracting 53″ from this time leaves the apparent time of the conjunction at Cambridge 11*h.* 36′ 20″·1. La Lande, by the mean of several observations made in Europe, found the conjunction at Paris to be at 4*h.* 30′ 6″ apparent time, as appears by the account of this eclipse published in his history of Astronomy for 1806, and in the Connoisance du Tems. The difference between these times of conjunction decreased by 9′ 21″ gives the difference of meridians of Greenwich and Cambridge 4*h.* 44′ 24″·9 used in the former part of this memoir.

The ecliptic conjunction at Paris 4*h.* 30′ 6″, corresponds to 4*h.* 20′ 50″·5 mean time at Greenwich. At this time, by the above elements, the sun's longitude was 84° 44′ 38″·4, the moon's longitude 84° 45′ 05″·5, the moon's latitude 19′ 17″·2 N. Hence if the sun's longitude be given correctly by Delambre's new tables, the error of Burg's tables in the longitude of the moon at that time was — 27″·1, and as the moon's true latitude was 19′ 19″·3, by page 28 of my former memoir, the error of Burg's tables in latitude was + 2″·1.

If from the times of conjunction calculated in pages 30, 31, and 32 of my former memoir, be subtracted the equation of time used there 6″·7, the remainder will be the apparent times of conjunction at those places, which subtracted from the apparent time of conjunction at Greenwich 4*h.* 20′ 45″, will give the corrected longitudes counted from the meridian of that Observatory. The effect of this operation

is to decrease by one second of time the longitudes given in those pages. The longitudes counted from Salem must also be decreased by 4″·7 more, on account of the difference between the longitude of Salem calculated in this memoir 4*h*. 43′ 36″·7 and that formerly used 4*h*. 43′ 32″.

Hence the longitude of Rutland calculated in page 30 ought to be 4*h*. 51′ 52″ from Greenwich: that of Kinderhook, page 31, 4*h*. 55′ 9″·6, and that of Philadelphia, page 32, 5*h*. 0′ 34″·7. Another observation was made at Philadelphia, by Professor Hassler, in a place in the latitude of 39° 57′ 2″, and 7″ in time W from the State House.

	App. time.	Par. in long.	Par. in lat.	☽ Aug. S. D.
Beginning	9*h*. 39′ 48″·5	25′ 11″·4	21′ 15″·7	16′ 39″·11
End	0 25 48 ·9	—5 19 ·2	16 52 ·8	16 42 ·08

Hence the apparent time of conjunction was 11*h*. 20′ 00″·7, whence the longitude of the State House is 5*h*. 00′ 37″·3. The mean of this and the other observation is 5*h*. 0′ 36″. Other observations made at this place will be given hereafter.

The apparent times of the beginning and end of total darkness, as observed at Albany, are given by Mr. De Witt in page 302, vol. vi. of the Transactions of the American Philosophical Society, at 11*h*. 8′ 6″ and 11*h*. 12′ 57″. Mr. Ferrer in the same volume, page 294, states that the equation of equal altitudes had been neglected in regulating the time-keeper, and the corrected observations are 11*h*. 8′ 14″·6 and 11*h*. 13′ 05″·6 mean time, being 2″ more than in page 31 of my memoir. This decreases the parallax in longitude nearly 0″·4, increases the time of conjunction at Albany 1″·3, and decreases the longitude 1″·3. Hence the whole decrement of longitude from Greenwich is 2″·3, making the longitude of Albany 4*h*. 54′ 59″·3 W from Greenwich.

From Mr. Ellicott's memoir in vol. vi. page 255, of the Transac-

tions of the American Philosophical Society, that the apparent times of the beginning and end of the eclipse at Lancaster were 9*h*. 53′ 8″ [erroneously printed 9*h*. 33′ 8″] and 0*h*. 18′ 56″. The mean times, used in page 32, are these quantities increased by 6″; whereas the former ought to have been increased 5″·8, the latter 7″·3. This increases the time of conjunction at Lancaster, given in page 32, nearly 0″·3. Hence the corrected longitude from Greenwich is 5*h*. 5′ 22″·2.

The observations at Natchez given by Mr. Dunbar in page 263, vol. vi. of the same Transactions, are, beginning 20*h*. 5′ 24″·6, end 22*h*. 38′ 54″·67 mean time, or 20*h*. 5′ 19″ and 22*h*. 38′ 47″·72 apparent time, which differ a few tenths of a second from those used in page 32 of my memoir. It is however to be observed, that Mr. Dunbar subtracted 5″ from the observed time of the beginning, supposing that time necessary to make the impression visible in the telescope; but as this correction is not applied to the other observations with which it is compared, it leads to an erroneous estimate of the longitude of the place. I assumed therefore 20*h*. 5′ 24″·1 and 22*h*. 38′ 47″·7 for the apparent times of observation, and using the elements in page 22, corrected for the errors of the longitude and latitude of the moon, found that the apparent time of conjunction became 10*h*. 15′ 15″·2 A. M. Hence the longitude of Mr. Dunbar's observatory is 6*h*. 5′ 29″·8, and the Castle of Natchez (supposed 9″ W) is 6*h*. 5′ 38″·8 W from Greenwich.

In page 276 of the sixth volume of the Transactions of the American Philosophical Society, are given the observations of this eclipse made at Bowdoin College, Brunswick, in latitude 43° 53′ N, reduced 43° 41′ 32″. The beginning was at 10*h*. 14′ 0″, the end at 12*h*. 55′ 20″ apparent time, and by using the elements of page 22, corrected as abovementioned, we have

	☽–☉ Par. long.	Par. lat.	☽ aug. sem.	App. time conjunction.	Longitude W Greenwich.
Beginning	17′ 57″·0	23′ 24″·8	16′ 39″·81	11*h*. 42′ 02″.3	4*h*. 38′ 42″·7
End	—10 42 ·3	21 02 ·4	16 41 ·49	11 41 07 ·7	4 39 37 ·3

The difference of nearly a minute, in the times of the conjunction deduced from the two contacts, arises probably from some mistake in noting the time of the beginning of the eclipse, since the end gives nearly the såme result as the eclipse of the sun of Sept. 17, 1811, as was mentioned in the former part of this paper.

Mr. Ferrer in a letter to President Webber, mentions that the end of the eclipse at Williamsburg, (Virginia) was at 0*h*. 15′ 14″ mean time. This would make the longitude 5*h*. 7′ 46″, which being nearly a minute more than other observations make it, and no account having been given of the observation by which we might judge of its accuracy, I have concluded to reject it.

Transit of Mercury of Nov. 9, 1769.

This transit was observed in Philadelphia, Norriton, and Salem. It was invisible in Europe. The difference of meridians of Philadelphia and Norriton is known to be 52 seconds in time, by a trigonometrical survey, and the observations made there, compared with those at Salem, would have given the difference of meridians of Salem and Philadelphia, but on calculating the observation at Salem, (which was made at a place 1″·2 east from the place where my observations of the eclipse of Sept. 17, 1811 were made) I found the times were too great by above a minute, owing probably to the inaccurate method of regulating the watch by a common meridian line. This rendered the observation useless.

Deductions from the eclipse of the sun, of October 27, 1780.

The observations of this eclipse, made at Chelsea, Beverly, Newport, Providence, Long-island, and Charlotte-town, are given in the

first volume of the Memoirs of the American Academy of Arts and Sciences. The elements of the eclipse for the times of observation at Chelsea, calculated by the Delambre and Burg are as follows.

	Beginning.	End.
	h. ′ ″	h. ′ ″
Apparent times of observation at Chelsea -	23 00 58	1 40 37
Mean times of observation at Chelsea -	22 44 56·8	1 24 35·3
Longitude west from Greenwich - -	4 44 03·7	4 44 03·7
Mean times of observation reduced to Greenwich	3 29 00·5	6 08 39·0
☉'s longitude counted from the apparent equinox	214° 50 43·5	214° 57 23·2
☉'s horizontal parallax - - -	8·87	8·87
☉'s semidiameter — Irradiation 3″·5 -	16 05·15	16 05·17
☉'s horary motion - - -	2 30·22	2 30·22
Apparent obliquity of the ecliptic - -	23 28 14	
☽'s longitude counted from the apparent equinox	213 53 44·0	215 30 33·7
☽'s latitude north decreasing - -	54 15·2	45 25·0
☽'s horizontal equatorial parallax - -	60 01·9	59 58·6
☽'s tabular semidiameter — inflexion 2″ -	16 21·07	16 20·17
☽'s horary motion in longitude - - -	36 25·61	36 21·46
☽'s horary motion in latitude - -	3 19·03	3 19·54
☽'s horary motion from the sun in longitude	h. 33 55·39	h. 33 51·24
☉'s right ascension - - -	14 10 14·9	14 10 40·7

None of the above places of observation were well situated to find with accuracy the error of the moon's latitude. However by various calculations, the result of which will be given hereafter, it was found that the correction to be applied to the moon's tabular latitude was — 10″·4, nearly ; and the error of the tabular longitude — 5″·7, supposing the sun's longitude to be correct. These corrections being made, the observations were re-calculated as follows.

Observations at Chelsea and Beverly, October 27, 1780.

The observations at Chelsea were made by the Rev. Mr. Payson. In the latitude of 42° 25′ 11″ N reduced 42°13′45″, and 26″ in time E from Cambridge, corresponding to 4*h.* 44′ 03″·7 W from Greenwich, the reduction of parallax being 5″·44.

	App. time.	(☽−☉) Par. lon.	(☽−☉) Par. lat.	☽ Aug. S. D.
Begin'g at Chelsea	23*h*. 00′ 58″	+26′ 44″·7	42′ 08″·3	16′ 30″·68
End	1 40 37	—1 16 ·1	51 47 ·7	16 28 ·89

Hence the apparent time of conjunction at Chelsea, was 0*h*. 41′ 55″·1, which added to 4*h*. 44′ 3″·7 gives the corresponding time at Greenwich 5*h*. 25′ 58″·8.

The observations at Beverly were made by the Rev. President Willard, Doctor Prince, and Doctor Cutler. The beginning was observed by them at 11*h*. 1′ 48″, 11*h*. 1′ 46″, and 11*h*. 1′ 42″. The end at 1*h*. 41′ 26″, 1*h*. 41′ 29″, and 1*h*. 41′ 23″ respectively. The mean of these times was used. The place of observation was found by trigonometrical survey to be 1′ 43″ N and 5″·6 in time E from the place in Salem, where I observed the eclipse of Sept. 17, 1811. Hence the latitude of the place is 42° 35′ 13″, reduced 42° 23′ 47″, longitude 58″·6 E from Cambridge, corresponding to 4*h*. 43′ 31″·1 W from Greenwich.

	(☽−☉) Par. long.	(☽−☉) Par. lat.	☽ S. D. Aug.
Beginning at Beverly	+26′ 36″·9	42′ 18″·2	16′ 30″·65
End	—1 19 ·7	51 54 ·5	16 28 ·84

Hence the apparent time of conjunction at Beverly was 0*h*. 42′ 38″·3, corresponding to 5*h*. 26′ 09″·4 at Greenwich. The mean of this and the observation at Chelsea gives the conjunction at Greenwich at 5*h*. 26′ 04″·1 apparent time, which is to be used in finding the longitudes of the other places of observation.

If the moon's corrected latitude used in this calculation was too small by *l* seconds, the time of conjunction at Greenwich would be decreased by 0·53 *l* seconds of time.

Observations at Long-Island, Penobscot, Oct. 27, 1780.

These observations were made by Doctor Williams and his assistants near the house of Mr. S. Williams, in a cove on the eastern

part of Long Island in Penobscot River (which appears to be called Williams's carrying-place in Hollond's chart) in the latitude of 44° 17′ 07″·26, reduced 44° 05′ 39″, the reduction of the moon's equatorial parallax being 5″·83. Messrs. Williams, Winthrop, and Atkins observed both contacts with the largest telescopes. The beginning, as observed by them respectively, was at 11*h*. 11′ 8″, 11*h*. 11′ 38″, 11*h*. 11′ 13″; the end at 1*h*. 50′ 25″, 1*h*. 50′ 17″ or 19″, and 1*h*. 50′ 28″ apparent times. The mean of these times was used.

	App. time.	(☽ − ☉) Par. long.	(☽ − ☉) Par. lat.	☽ aug. S. D.
Beginning	11*h*. 11′ 19″·7	+25′ 02″·8	44′ 01″·6	16′ 30″·34
End	1 50 24 ·0	—1 59 ·7	53 01 ·4	16 28 ·21

Hence the apparent time of conjunction was at 0*h*. 50′ 44″·0, which, subtracted from the time of conjunction at Greenwich 5*h*. 26′ 04″·1, leaves the longitude of the place of observation on Long-Island 4*h*. 35′ 20″·1 W from Greenwich.

An increase of *l* seconds in the moon's latitude would decrease the time of conjunction at Long Island 0.52 *l* seconds, which subtracted from the corresponding correction at Beverly and Chelsea 0·53 *l* leaves the error of the longitude of Long-Island — 0·01 *l*, arising from an increase of *l* seconds in the moon's latitude. Hence it is evident that an error of 10 or 12 seconds in this element, will not sensibly affect the longitude of that place, calculated by the above observation.

At Charlotte Town, Island of St. John, Gulf of St. Lawrence, Oct. 27, 1780.

Observations were made at this place, by Messrs. Clarke and Wright in the latitude of 46° 13′ reduced 46° 1′ 32″, reduction of the equatorial parallax 6″·23.

	App. time.	(☽ − ☉) Par. long.	(☽ − ☉) Par. lat.	☽ aug. S.D.
Beginning	11*h*. 41′ 35″	+20′ 04″·6	47′ 09″·2	16′ 30″·02
End	2 17 41	—5 13 ·6	54 42 ·2	16 27 ·01

Hence the apparent time of conjunction was at 1*h.* 13′ 33″·0, which subtracted from the time of conjunction at Greenwich leaves the longitude of the place of observation 4*h.* 12′ 31″·1.

An increase of l seconds in the moon's latitude, would decrease the time of conjunction 0·46 l seconds of time, which subtracted from the corresponding correction at Beverly and Chelsea leaves ·07 l, which is the number of seconds of time to be subtracted from the longitude of Charlotte-town for an increase of l seconds in the moon's latitude, and as this correction is very small, there can be but little uncertainty in the longitude on this account.

Observations at Newport and Providence Oct. 27, 1780.

The beginning of the eclipse was not observed at these places. The end was observed at Newport, R. I. by Mr. Granchain, at 1*h.* 40′ 41″ apparent time. He states the latitude of the place to be 41° 30′ 30″. Hollond makes it 41° 28′ 28″. The mean is nearly 41° 29½′* which used as the true latitude makes the reduced latitude 41° 18′ 7″, the reduction of the moon's equatorial parallax 5″·24, the (☽—☉) par. in long.—1′ 43″·2, in latitude 51′ 21″·3, ☽'s augmented semidiameter 16′ 29″·09. Hence the conjunctions at Newport was at 0*h.* 40′ 58″·7 apparent time. The difference of meridians of Newport and Providence according to Hollond's survey is 18″·8, which would make the conjunction at Providence by this observation 0*h.* 40′ 39″ 9.

The end of the eclipse at Providence, by the observations of Messrs. Brown and West was at 1*h.* 39′ 08″·3 apparent time. The latitude of the place by Hollond is 41° 48′ 50″, reduced 41° 37′ 26″.

* This difference in the latitude is scarcely sensible in the computation of the longitude from this observation, since an increase of one minute in the latitude decreases the longitude but half a second of time. The same is to be observed of the observation at Providence.

The (☽—☉) par. in long.—1′ 18″·0, in latitude 51′ 26″·8, ☽'s augmented semidiameter 16′ 29″·06. Hence the conjunction at Providence by this observation was at 0*h.* 40′ 11″·2. The mean of this and the former result 0*h.* 40′ 39″·9 is 0*h.* 40′ 25″·5, which may be assumed as the apparent time of conjunction at Providence. This, subtracted from the time of conjunction at Greenwich, gives the longitude of Providence 4*h.* 45′ 38″·6, and by subtracting 18″·8, the longitude of Goat Island, Newport, is obtained 4*h.* 45′ 19″·8 W from Greenwich, which agrees nearly with the observations of the transit of Venus of June 3, 1769.

The longitudes of these two places are liable to a greater error from the uncertainty of the moon's latitude, than would have been the case if both contacts had been observed. For an increase of l seconds in the moon's latitude would decrease the times of conjunction at these places by 0·33 l seconds, which subtracted from the corresponding change in the time of conjunction at Greenwich 0″·53 l, leaves the variation of the longitudes just calculated 0″·2 l. Hence an increase of 5 seconds in the moon's latitude (corrected as above) would decrease the longitudes of Newport and Providence by one second of time, nearly, and the error from this source does not probably exceed that quantity.

At the apparent time of the conjunction at Greenwich 5*h.* 26′ 04″·1, the moon's tabular longitude was 214° 55′ 02″·1, the sun's longitude 214° 54′ 56″·4, the difference — 5″·7 is the correction to be applied to the longitude of the moon given by Burg, supposing the solar tables to be correct. This is the same as was assumed at the commencement of the calculation.

To find the error in the moon's latitude, the observations at Long Island, Beverly, and Providence were used as follows.

The greatest obscuration at Long Island was at 0*h.* 31′ 18″ apparent time, when an arch of 42° or 43° of the sun's disc was visible. Supposing this arch to be 42° 30′, the sun's semidiameter 16′ 05″·16, the moon's augmented semidiameter 16′ 29″·97, it is easy to find that the apparent distance of the centres of the sun and moon was 26·57, observing that the diameter, passing through those centres, bisects perpendicularly the chord connecting the extreme visible points of the sun's limb. At this time the (☽—☉) parallax in longitude was 11′ 20″·0, in latitude 49′ 13″·6, the apparent difference of longitudes of the centres of the sun and moon, found from the tables correcting the moon's longitude for the error — 5″·7 was 21″·52. From the apparent distance of the centres and their apparent difference of longitude, it is easy to find the apparent difference of latitude 15″·6, which added to the parallax in latitude, gives the latitude by this observation 42′ 29″·2; this subtracted from the tabular latitude 49′ 44″·3 gives the error in latitude by this observation — 15″·2.

At the same place at 0*h.* 28′ 48″ the lucid part of the sun when least, measured by a micrometer, was 24″.7. This added to the ☽'s augmented semidiameter 16′ 29″·99 and the ☉'s semidiameter subtracted from the sum, gives the apparent distance of the centres of the sun and moon 49″·53. The (☽—☉) par. in long. was 11′ 46″·0, and the apparent difference of longitude (found as above) was 37″·1, consequently the apparent difference of latitude 32″·8, which added to the parallax in latitude 49′ 04″·9, gives the latitude by observation 49′ 37″·7, which subtracted from the tabular latitude 49′ 52″·6 leaves the error of the tables in latitude by this observation — 14″·9.

At Beverly at 0*h.* 21′ apparent time the sun was eclipsed $11^{d.}$ 24′, which makes the lucid part 96″·52. The sun's semidiameter was then 16′ 05″·16, the moon's augmented semidiameter 16′ 30″·47, the (☽—☉) paral. in longitude was 12′ 43″·5, paral. in latitude 47′ 44·″0,

and, by a similar calculation to the preceding, the error of the moon's latitude by this observation is — 10″·1.

At Providence the greatest obscuration was nearly at 0*h.* 18′ 55″ apparent time, the lucid part measured by a micrometer was then $\frac{280}{3868}$, parts of the sun's semidiameter, equal to 139″·73. The sun's semidiameter 16′ 05″·16, the moon's augmented semidiameter 16′ 30″·67, the (☽—☉) parallax in longitude 12′ 55″·0, in latitude 47′ 11″·0. Hence the error in latitude by this observation is — 1″·4.

The mean of these four observations gives the error of latitude — 10″·4, the same as was assumed.

Eclipse of the sun of June 24, 1778.

The end of this eclipse was observed at Bradford, Massachusetts, by Doctor Williams at 11*h.* 38′ 16″ A. M. apparent time. The latitude of the place is nearly 42° 46′ N, reduced 42° 34′ 34″. Using the same tables of Delambre and Burg, correcting the moon's longitude + 6″·2 and latitude + 1″·1, as deduced from the observations at Greenwich before given; supposing the irradiation 3″·5, inflection 2″. It was found that the (☽—☉) parallax in longitude was + 5′ 14″·7, in latitude 19′ 52″·1, whence the apparent time of conjunction at Bradford was at 10*h.* 51′ 46″·2 A. M. apparent time. The difference between this and the time of conjunction at Greenwich, calculated before, 3*h.* 35′ 52″·9 gives the longitude of Bradford 4*h.* 44′ 6″·7 W from Greenwich, or 30″·0 W from Salem, which agrees very nearly with the map of Massachusetts.

Annular eclipse of April 3, 1791.

This eclipse was observed at Georgetown, Columbia, by Mr. Ellicott, in the latitude of about 38° 55′ N, reduced 38° 43′ 47″, as appears by vol. iv. page 48 of the Transactions of the American Philosophical Society, and the observations have been calculated by Mr.

Ferrer in the sixth volume of the same work. This gentleman supposes that the time of forming the annulus was marked too small by one minute, a mistake that might easily have been made, and of which several instances have occurred in the most important observations made by the best astronomers of Europe. This correction being applied, makes the times of conjunction, deduced from the three observations, agree much more nearly with each other than they otherwise would do. I have therefore adopted it in the following calculation. Using the tables of Delambre and Burg, correcting the moon's longitude by adding 20″, and the moon's latitude by adding 7″·6, which is necessary from the observation at Greenwich and Paris, mentioned in the former part of this memoir, the calculation becomes

	Mean time.	(☽−☉) Par. lon.	Par. lat.
Formation of Annulus	18*h*. 40′ 01″$\frac{1}{4}$	25′ 40″·7	46′ 54″·6
Breaking of Annulus	18 43 15 $\frac{1}{4}$	25 36 ·7	46 49 ·2
End of the eclipse	19 55 37 $\frac{3}{4}$	22 20 ·0	44 00 ·8

The conjunction by the mean of these three observations is at 19*h*. 37′ 00″·4 mean time, or 19*h*. 33′ 42″·6 apparent time. The difference between this and the conjunction at Greenwich, calculated before 0*h*. 42′ 00″·2, is 5*h*. 8′ 17″·6, the longitude of Georgetown, and as this place by the measurement of Seth Pease Esq. is 13·1 seconds, in time W from the Capitol in Washington, the longitude of the Capitol would be by this observation 5*h*. 8′ 4″·5. The observation of Mr. Pease on the eclipse of 1811, makes it in 5*h*. 8′ 11″·4. The mean of both gives the longitude of the Capitol in Washington 5*h*. 8′ 8″·0, whence the longitude of Georgetown is 5*h*. 8′ 21″·1 = 77° 5′ 16″ W from Greenwich.

The observations of this eclipse at Philadelphia, given by Mr. Rittenhouse in vol. iii. page 154 of the Transactions of the American Philosophical Society of that city, appear to be erroneous. For the

conjunction deduced from the fourth contact agrees nearly with other observations, but the three first contacts make the conjunction too late by nearly a minute. Mr. Ferrer in his calculation in vol. vi. of the same Transactions, rejects the first contact, and subtracts one minute from the second and third; but this correction does not appear to be warranted by the observations. For if we suppose the regulation of the time-keeper to be correct, there must have been a mistake in reading off the *three* first observations, which is improbable; and if we suppose the regulation of the time-keeper to be erroneous by one minute, it will not make the observations accord without allowing also an error of one minute in the last contact. These reasons have induced me to reject the observations, as not having been made with sufficient accuracy to be used in calculating the longitude of Philadelphia.

Transit of Venus June 3, 1769.

The elements of this transit for June 3, 1769, at 10*h.* 2′ 34″·1 mean time at Greenwich, are as follows.

☉'s longitude by Delambre's tables of 1806 - -	73° 27′ 18″·9
♀'s longitude } deduced from the observations at Greenwich, Paris, and Prince of Wales' Fort.	73 27 18 ·9
♀'s latitude }	10 14 ·32 N.
☉'s horary motion - - - - -	143 ·46
♀'s horary motion in longitude - - - -	94 ·144
♀'s horary motion in latitude - - - -	—35 ·450
♀'s horizontal parallax - - - - -	30 ·45
☉'s horizontal parallax - - - - -	8 ·67
☉'s semidiameter by T. 2. P. 470 of La Lande's Astronomy	15 43 ·71
♀'s semidiameter - - - - - -	28 ·60
☉'s right ascension - - - - -	4*h.* 48 13 ·23
Horary increment of right ascension - - -	10 ·27

The mean of the observations of Messrs. Maskelyne, Dollond, Nairn, Hirst, Hitchins, Horsley, and Dunn, at the Royal Observatory at Greenwich, as given in the first volume of the Transactions of the Philosophical Society of Philadelphia, make the first external contact at 7*h.* 11′ 1″·9 and the first internal contact at 7*h.* 29′ 19″·1. The

observation of the internal contact by Mr. Hirst is in one place printed $7h.\ 28'\ 57''$, in another $7h.\ 28'\ 47''$; that of Mr. Dunn $7h.\ 29'\ 28''$ and $7h.\ 29'\ 48''$. The values $7h.\ 28'\ 57''$, and $7h.\ 29'\ 28''$, were used as best agreeing with the other observations. The mean of the observations of the first internal contact at Paris, made by Messrs. Messier, Cassini, Sejour, Chaulnes, Maraldi, Fouchy, and Bery, as published in the Memoirs of the Royal Academy of Arts and Sciences of Paris, was $7h.\ 38'\ 48''{\cdot}7$ apparent time. The parallaxes and times of conjunction are as follows.

	(♀−☉) Par. long.	Par. lat.	Conjunction.	Conj. Green.
I. contact Greenwich	—15″·000	+15″·551	10*h*. 4′ 49″·6	10*h*. 4′ 49″·6
II. contact Greenwich	—14 ·681	+15 ·988	10 4 44 ·3	10 4 44 ·3
II. contact Paris	—15 ·120	+15 ·658	10 14 16 ·2	10 4 55 ·2
The mean of these three observations gives the apparent time of the conjunction at Greenwich.				10 4 49 ·7

The mean of the observations of Messrs. Dymond and Wales, at Prince of Wales' Fort in Hudson's Bay, in latitude 58° 47′ 32″, were as below.

	App. time.	♀−☉ Par. lon.	Par. lat.	Conj. app. time.	Lon. W Green.
I. contact	0*h*. 57′ 04″·1	−4″·322	+12″·491	3*h*. 47′ 49″·6	6*h*. 17′ 00″·1
II. contact	1 15 23 ·3	−5 ·202	+12 ·483	54 ·8	6 16 54 ·9
III. contact	7 00 47 ·0	−13 ·110	+16 ·841	55 ·0	6 16 54 ·7
IV. contact	7 19 10 ·¾	−12 ·876	+17 ·206	49 ·9	6 16 59 ·8
				Mean	6 16 57 ·4

The internal contacts at this place were used in finding the latitude of Venus at the time of conjunction inserted in the above elements.

Professor Winthrop at Cambridge missed the first contact, but had a good observation of the second contact at $2h.\ 47'\ 30''$ apparent time. The parallax in longitude was then — $11''{\cdot}743$, in latitude $+7''{\cdot}484$. Hence the apparent time of the conjunction by this obser-

vation was 5*h*. 20′ 8″·7. Professor Williams observed the same contact at Newbury in latitude 42° 48′ N, and by Hollond's survey 6½ seconds in time E from my house in Salem, or 59½ seconds E from Cambridge, at 2*h*. 48′ 44″ apparent time. The parallax in longitude was then — 11″·757, in latitude + 7″·643, whence the conjunction at Newbury was at 5*h*. 21′ 28″·0, corresponding to 5*h*. 20′ 28″·5 apparent time at Cambridge. The mean of this and the former result is 5*h*. 20′ 18″·6, which subtracted from the time of conjunction at Greenwich 10*h*. 4′ 49″·7, gives the longitude of Cambridge by this observation 4*h*. 44′ 31″·1.

At Providence in latitude 41° 50′ 41″, the transit was observed by Messrs. West and Brown. The first contact was not well observed, but the second contact was at 2*h*. 46′ 35″ apparent time. The parallax in longitude was — 11″·762, in latitude + 7″·282. Hence the conjunction was at 5*h*. 19′ 12″·7 apparent time, which subtracted from the time of conjunction at Greenwich 10*h*. 4′ 49″·7 gives the longitude of Providence by this observation 4*h*. 45′ 37″·0. By the eclipse of Oct. 27, 1780 it was 4*h*. 45′ 38″·6. The mean gives the longitude of Providence 4*h*. 45′ 37″·8, and by subtracting 18″·8 we have the longitude of Goat Island, Newport, 4*h*. 45′ 19″·0 W from Greenwich.

At the State House square in Philadelphia, in the latitude of 39° 57′ 10″, the transit was observed by Messrs. Shippen, Williamson, Prior, Ewing, Pearson, and Thompson. At Norriton in the latitude of 40° 9′ 56″, and by an accurate terrestrial measurement 52″ in time W from Philadelphia, the transit was observed by Messrs. Smith, Lukens, and Rittenhouse. At Lewistown in the latitude of 38° 46′ 38″ and 1″·4 in time E from Philadelphia, as determined by an accurate survey, the transit was observed by Messrs. Biddle and Bayley. The detail of these observations is given in the first volume of the Trans-

actions of the Philosophical Society held at Philadelphia. The times of conjunction calculated at these places from the mean of the observations, and reduced to the meridian of the State House in Philadelphia, are as follows.

		App. time.	Par. long.	Par. lat.	Conj. Philadel. App. time.
Philadelphia	I. contact	2*h*. 13′ 46″·6	—10″·030	6″·287	5*h*. 4′ 23″·4
	II. contact	2 31 28 ·4	—11 ·115	6 ·447	3 40 ·7
Norriton	I. contact	2 12 57 ·8	— 9 ·960	6 ·357	4 27 ·2
	II. contact	2 30 07 ·0	—11 ·012	6 ·510	3 05 ·7
Lewistown	I. contact	2 14 19 ·0	—10 ·167	5 ·865	4 54 ·6
	II. contact	2 32 08 ·0	—11 ·277	6 ·030	4 20 ·6

The mean of these six observations gives the apparent time of conjunction at Philadelphia 5*h*. 4′ 8″·7, which subtracted from 10*h*. 4′ 49″·7 gives the longitude of Philadelphia by this observation 5*h*. 0′ 41″·0.

The transit was observed at Baskenridge, New Jersey, in the latitude of 40° 40′ N, by the Earl of Sterling, as follows.

	App. time.	Par. long.	Par. lat.	Conj. App. time.
I. contact	2*h*. 16′ 00″	—10″·102	6″·562	5*h*. 6′ 39″·9
II. contact	2 34 12	—11 ·199	6 ·729	5 6 33 ·3

The mean of these observations makes the conjunction at 5*h*. 6′ 36″·6 apparent time, which subtracted from 10*h*. 4′ 49″·7 gives the longitude of Baskenridge 4*h*. 58′ 13″·1 W from Greenwich.

At Quebec, in the latitude of 46° 47′ 17″ N, and 10″·4 in time W from the Castle of St. Lewis, the first contact was observed by Mr. Hollond at 2*h*. 30′ 8″$\frac{3}{4}$ apparent time. The parallax in longitude was — 10″·236, in latitude + 8″·837, whence the apparent time of conjunction was 5*h*. 21′ 17″·6, and the longitude of the Castle of St. Lewis 4*h*. 43′ 21″·7. At the Island of Coudre, in the latitude of 47° 16′ 30″ N, and 5′ 6″ in time E from Quebec, the second contact was observed by Mr. Wright, at 2*h*. 50′ 50″ apparent time. The parallax in longitude was — 11″·242, in latitude 9″·208. Whence the appar-

ent time of conjunction was $5h.\ 23'\ 45''\cdot1$, which subtracted from $10h.\ 4'\ 49''\cdot7$, gives the longitude of that island $4h.\ 41'\ 04''\cdot6$, whence that of Quebec is $4h.\ 44'\ 10''\cdot6$. The mean of this and the former result is $4h.\ 43'\ 46''\cdot1$ for the longitude of Quebec, and $4h.\ 40'\ 40''\cdot1$ for that of I. Coudre.

Transit of Mercury of Nov. 12, 1782.

Observations of this transit were made at Paris, Cambridge, Philadelphia, New Haven, and Ipswich. Those made in this country were under more favourable circumstances than those at Paris, on account of the greater elevation of the sun above the horizon, but for the reasons before assigned, no great accuracy was to be expected in the longitudes found by comparing the American observations with each other; hence it was thought best not to use these observations.

Transit of Mercury of Nov. 5, 1789.

The observations at Cambridge are given in the second volume of the memoirs of the American Academy of Arts and Sciences, these at Philadelphia and Williamsburg in the second volume of the Transactions of the American Philosophical Society held at Philadelphia.

The elements of this transit for Nov. 5, 1789 at $3h.\ 7'\ 50''\cdot9$ mean time at Greenwich, are as follows.

☉'s longitude by Delambre's tables of 1806 app. equinox	$223°\ 40'\ 45''\cdot4$
☿'s longitude by La Lande's tables corrected by the observations at Cambridge	223 40 45 ·4
☿'s latitude south decreasing, - - -	7 15 ·1
☉'s horary motion - - - -	150 ·73
☿'s horary motion in longitude - -	199 ·03
☿'s horary motion in latitude - - -	51 ·78
☿'s horizontal parallax - - -	13 ·03
☉'s horizontal parallax - - -	8 ·89
☿'s semidiameter - - - -	4 ·664
☉'s semidiameter — irradiation $3''\cdot5$ - - -	16 07 ·26
Horary variation of the hor. mot. of ☿ in longitude -	— 0 ·061
- - - - in latitude -	+ 0 .015
☉'s right ascension - - -	$14h.$ 44 52 ·07
Horary increment of right ascension in time - - -	9 ·97

The observations at Cambridge give the following result.

	App. time.	☿−☉ Par. long.	Par. lat.	Conj. app. time.
I. contact	20*h*. 24′ 04″	3″·321	2″·282	22*h*. 39′ 30″·3
II. contact	20 25 52	3 ·309	2 ·292	31 ·9
III. contact	1 15 44	0 ·080	3 ·614	26 ·5
IV. contact	1 17 36	0 ·057	3 ·619	31 ·7

The mean of these four observations makes the conjunction at 22*h*. 39′ 30″·1, apparent time at Cambridge, corresponding to 3*h*. 23′ 59″·8 apparent time at Greenwich. I have preferred finding the conjunction at Greenwich from this observation rather than from that at Paris, on account of the failure in part of this observation, as mentioned in the preceding calculation of the longitude of Cambridge. The latitude of Mercury given in the above elements was decreased 0″·06, which appears to be necessary from the observations at Cambridge and Philadelphia.

The observations at Philadelphia were as follows.

	App. time.	(☿−☉) Par. long.	Par. lat.	Conj. app. time.
I. contact	20*h*. 7′ 59″·1	3″·481	2″·063	22*h*. 23′ 26″·1
II. contact	20 9 29 ·1	3 ·472	2 ·071	07 ·8
III. contact	0 59 32 ·6	0 ·225	3 ·485	13 ·2
IV. contact	1 01 12 ·6	0 ·203	3 ·490	7 ·0

The mean of these observations makes the apparent time of conjunction 22*h*. 23′ 13″·5. The difference between this and 3*h*. 23′ 59″·8 gives the longitude of Philadelphia by this observation 5*h*. 0′ 46″·3 W from Greenwich. The transit of Venus of June 3, 1769, made it 5*h*. 0′ 41″·0. The total eclipse of June 16, 1806 gave 5*h*. 0′ 36″. The mean of these three observations may be assumed as the longitude of Philadelphia 5*h*. 0′ 41″·1 W from Greenwich, which differs less than a second from the estimate of Mr. Ferrer in vol. vi. page 359 of the Transactions of the American Philosophical Society,

which is $5h.\ 10'\ 1''{\cdot}2$ W from Paris, corresponding to $5h.\ 0'\ 40''{\cdot}2$ W from Greenwich. Mr. Ferrer's calculation is made by the three observations just named, and those of the transit of Mercury of 1782 and the annular eclipse of 1791. These last observations I have rejected for the reasons stated in the former part of this memoir. Subtracting $1''{\cdot}4$ from the longitude of Philadelphia, gives the longitude of Lewistown $5h.\ 0'\ 39''{\cdot}7$, and by adding $52''$ the longitude of Norriton is obtained $5h.\ 1'\ 33''{\cdot}1$ W from Greenwich.

Messrs. Owen and Biddle found, by a trigonometrical survey, that the Light House on Cape Henlopen was $29''{\cdot}8$ N, and $13''{\cdot}1$ in time E from their observatory at Lewistown. Hence the Light House is in the latitude of $38°\ 47'\ 8''$ N, and in longitude $5h.\ 0'\ 26''{\cdot}6$ W from Greenwich.

The same transit of 1789 was observed at Williamsburg as follows.

	App. time.	(☿−☉) Par. long.	Par. lat.	App. time conj.
II. contact	$20h.\ 3'\ 10''$	$3''{\cdot}570$	$1''{\cdot}884$	$22h.\ 16'\ 48''{\cdot}0$
III. contact	0 53 45	0 ·239	3 ·368	17 23 ·9
IV. contact	0 55 10·5	0 ·219	3 ·372	17 03 ·9

The mean of these three observations makes the conjunction at $22h.\ 17'\ 5''{\cdot}3$, which subtracted from $3h.\ 23'\ 59''{\cdot}8$, gives the longitude of Williamsburg by this observation $5h.\ 6'\ 54''{\cdot}5$. The solar eclipse of Sept. 17, 1811 makes it $5h.\ 6'\ 48''{\cdot}4$. The mean of both observations may be assumed as the longitude of William and Mary College $5h.\ 6'\ 51''{\cdot}5$ W from Greenwich, which is about $52''$ less than the computation of Mr. Ferrer from the end of the solar eclipse of June 1806, an imperfect observation, which it was thought best to reject.

Eclipse of the sun of June 26, 1805.

The beginning of this eclipse was noted at Philadelphia, Lancaster, and New York. No observations were made to determine the error of the moon's tabular latitude, but it is probable from the following calculation, that this would not much affect the computed difference of meridians between those places, so that the longitude of New York may be obtained by these observations to a considerable degree of accuracy, by means of the longitudes of Philadelphia and Lancaster, computed in the former part of this memoir.

The observation at Philadelphia was made by Professor Patterson at 6*h.* 47′ 40″·5 apparent time, in the latitude of 39° 57′ 2″, longitude 5*h.* 0′ 41″·1 W from Greenwich. The corresponding apparent time at Greenwich was 11*h.* 48′ 21″·6, when by the tables of Delambre and Burg, the elements were as follows.

	° ′ ″
☉'s longitude from apparent equinox - - -	94 48 30·5
☽'s longitude - - - - - -	95 08 27·0
☽'s latitude north decreasing - - - -	62 23·0
☉'s horary motion in longitude - - -	2 23·0
☽'s horary motion in longitude - - - - -	37 58·41
☽'s horary motion in latitude - - - -	3 26·66
Horary decrement ☽ hor. mot. long. - - - -	0·74
Horary increment ☽ hor. mot. lat. - - - - -	0·46
(☽—☉) horizontal parallax for Philadelphia - -	61 04·72
☽'s semidiameter — inflexion 2″ + aug. 2″ - - -	16 43·95
☉'s semidiameter — Irradiation 3″·5 - - -	15 42·05
☉'s right ascension - - - - -	6*h.* 20 57·5
(☽—☉) Par. in long. - - - - - -	—44 31·1
(☽—☉) Par. in lat. - - - - -	41 18·7

Hence the conjunction at Philadelphia was at 6*h.* 14′ 11″·6, corresponding to 11*h.* 14′ 52″·7 apparent time at Greenwich. An increase of 1″ in the moon's latitude would decrease this 1″·44. Hence if the correction to be added to the moon's tabular latitude be *l* seconds, the apparent time at Greenwich will be at 11*h.* 14′ 52″·7 — 1″·44·*l*, by this observation.

The observation at Lancaster, in the latitude of $40° 2' 36''$, longitude $5h.\ 5' 22''\cdot 2$ W from Greenwich, was made by Mr. Ellicott at $6h.\ 43' 26''$ apparent time. At that time by the above elements the moon's augmented semidiameter was $16' 44''\cdot 18$, (☽—☉) horizontal parallax $61' 4''\cdot 70$, par. in long. $-44' 38''\cdot 5$, par. in lat. $41' 2''\cdot 2$. Hence the apparent time of conjunction was $6h.\ 9' 23''\cdot 3$, corresponding to $11h.\ 14' 45''\cdot 5$ at Greenwich, and, by increasing the moon's latitude l seconds, this would become $11h.\ 14' 45''\cdot 5 - 1''\cdot 46\cdot l$. The mean of this and the former value is $11h.\ 14' 49''\cdot 1 - 1''\cdot 45\cdot l$.

The observation at New York was made by Mr. Ferrer in latitude of $40° 42' 40''$ N, at $6h.\ 50' 10''$ apparent time. The moon's augmented semidiameter was $16' 43''\cdot 92$, (☽—☉) horizontal parallax $61' 4''\cdot 56$, par. in long. $- 43' 52''\cdot 4$, par. in lat. $42' 0''\cdot 5$. Hence the apparent time of conjunction was $6h.\ 18' 34''$, and by increasing the moon's latitude by l seconds, this becomes $6h.\ 18' 34'' - 1''\cdot 38\cdot l$, which subtracted from the time of conjunction at Greenwich $11h.\ 14' 49''\cdot 1 - 1\cdot 45\ l$, gives the longitude of the place of observation $4h.\ 16' 15''\cdot 1 - 0''\cdot 07\ l$, and as this place is $0''\cdot 3$ W of Columbia College, the longitude of that College would be $4h.\ 56' 14''\cdot 8 - 0''\cdot 07\ l$, in which the coefficient of l is small, and we may without much error assume the longitude to be $4h.\ 56' 14''\cdot 8$ by this observation. This was combined with other observations in the former part of this memoir.

The longitudes here calculated, with the addition of a few places in the vicinity of Salem, marked with an asterisk, which I have found by a trigonometrical survey, are collected in the following table. The longitudes of Boston State-House and Light-House from Cambridge were found by Professor Farrar.

Most of the preceding calculations have been made in two different ways, to verify the accuracy of the results.

Table of Latitudes and Longitudes.

	Latitudes North.	Longitudes W from Greenwich.	
	° ′ ″	h. ′ ″	° ′ ″
Albany, New York - - - - -	42 38 39	4 54 59·3	73 44 49
* Baker's island light houses - - -	42 34 10	4 43 09·8	70 47 27
Baskenridge - - - - -	40 40	4 58 13·1	74 33 16
* Beverly, President Willard's house - -	42 35 13	4 43 31·1	70 52 46
Boston, State House - - - -	42 22 28	4 44 16·6	71 04 09
——— Light House - - - -	42 20 41	4 43 35·0	70 53 45
Bradford, Massachusetts - - -		4 44 06·7	71 01 40
Brunswick, Bowdoin College - - -	43 53	4 39 40·1	69 55 01
Burlington College, Vermont - -	44 28	4 52 58·3	73 14 34
Cambridge, Harvard Hall - - -	42 23 28	4 44 29·7	71 07 25
Cape Henlopen - - - -	38 47 08	5 00 26·6	75 06 39
Charlottetown, St. John's I. - - -		4 12 31·1	63 07 46
Chelsea - - - - -	42 25 11	4 44 03·7	71 00 55
Coudre Island - - - -	47 16 30	4 40 40·1	70 10 01
Georgetown - - - - -	38 55	5 08 21·1	77 05 16
Kinderhook - - - - -	42 23 08	4 55 09·6	73 47 24
Lancaster - - - - -	40 02 36	5 05 22·2	76 20 33
Lewistown - - - - -	38 46 38	5 00 39·7	75 09 55
Long Island, Penobscot - - -	44 17 07	4 35 20·1	68 50 01
* Lynn, Phillip's Point - - -	42 30 14	4 43 34·9	70 53 43
* Marblehead, West meeting house - -	42 32 30	4 43 25·8	70 51 27
* Manchester, Glass Head - - -	42 35 42	4 43 08·5	70 47 07
Monticello - - - - -	38 08	5 15 10·4	78 47 36
Nantucket, near middle of town - -	41 15 32	4 40 31·6	70 07 54
Natchez, Mr. Dunbar's observatory - -	31 27 48	6 05 29·8	91 22 27
———— Castle - - -		6 05 38·8	91 24 42
New Brunswick, New Jersey, Columbia College	40 29 34	4 57 54·1	74 28 31
New Haven, Yale College - - -	41 17 58	4 51 51·1	72 57 46
Newport, Goat Island - - - -		4 45 19·0	71 19 45
New York, Columbia College - - -	40 42 45	4 56 00·2	74 00 03
Norriton - - - - - -	40 09 56	5 01 33·1	75 23 16
Philadelphia - - - - -	39 57 02	5 00 41·1	75 10 16
Portland, Observatory Hill - - -	43 39	4 41 22	70 20 30
Prince of Wales' Fort, Hudson's Bay - -	58 47 32	6 16 57·4	94 14 21
Providence - - - - - -	41 50 41	4 45 37·8	71 24 27
Quebec - - - - - -		4 43 46·1	70 56 31
Rutland, Vermont - - - - -	43 36	4 51 49·8	72 57 27
Salem, place of observing the eclipses of 1806, & 1811	42 33 30	4 43 36·7	70 54 10
——— Mr. Lambert's place of observation -	42 33 36	4 43 34·0	70 53 30
Washington City, Capitol - - - -	38 53 37	5 08 08·0	77 02 00
Williamsburg College - - - -	37 15 20	5 06 51·5	76 42 52

On the Altitude and Longitude of the Nonagesimal degree of the ecliptic.

The altitude and longitude of the nonagesimal degree of the ecliptic, necessary in the calculation of the parallaxes in the preceding observations, were found by the method I published in the third edition of the Practical Navigator, and as this is shorter and not liable to so many cases as that given in the first volume of the Memoirs of the Academy, I have thought that an explanation and demonstration of the method would not be unacceptable.

The method is considerably abridged by means of a table containing the logarithms marked A, B, C, which occur twice in calculating a partial eclipse of the sun or occultation, and four times in a total or annular eclipse or transit. These logarithms are calculated for the obliquity of the ecliptic 23° 27′ 40″, by the following rule.

In north latitudes subtract the reduced latitude from 90°, in south latitudes add the reduced latitude to 90°, the sum or difference will be the polar distance; take half this and half of the obliquity of the ecliptic (11° 43′ 50″), and find the sum S and the difference D. Then

$$\text{Log. } A = \text{Log. Cos. } D + \text{Log. Sec. } S - 20.$$
$$\text{Log. } C = \text{Log. Tang. } S.$$
$$\text{Log. } B = \text{Log. Tang. } D - \text{Log. } C + 10.$$

Thus for Salem in the reduced latitude 42° 22′ 4″, the half polar distance is 23° 48′ 58″, the half obliquity 11° 43′ 50″, the difference D = 12° 5′ 8″, the sum S = 35° 32′ 48″.

Difference	D 12° 5′ 8″	Cosine 9·9902660	Tang. + 10 =	19·3306527
Sum	S 35 32 48	Secant 10·0895665	Tang. = C =	9·8540160
	Sum	A = 0·0798325	Diff. B =	9·4766367

In this way the logarithms may be found for places not included in the table. The changes for an increase of 100″ in the latitude or

obliquity are found by repeating the operation with the increased values, and ascertaining the corresponding changes in the values of A, B, C. These logarithms are given to six places of figures, though in general five will be quite sufficient, since the latitude and longitude of the nonagesimal degree are rarely required to a greater degree of accuracy than 10″.

TABLE.

Places.	Reduced Latitude North.	Log. A	Var. A +100″ lat	Var. A +100″ obl.	Log. B	Var. B +100″ lat.	Var. B +100″ obl.	Log. C	Var. C +100″ lat.	Var. C +100″ obl.
	° ′ ″		—	+		—	—		—	+
Albany	42 27 13	0 079670	53	97	9 475783	293	739	9 853328	223	223
Berlin	52 20 24	0 061608	49	75	9 324135	618	1099	9 771197	240	240
Beverly	42 23 47	0 079778	53	98	9 476336	292	737	9 853787	223	223
Boston	42 10 46	0 080190	53	98	9 478602	288	733	9 855524	222	222
Brunswick Coll.	43 41 32	0 077334	52	95	9 462117	218	767	9 843361	224	224
Burlington Coll.	44 16 32	0 076242	52	93	9 455341	330	781	9 838640	225	225
Cambridge (Eng.)	52 01 28	0 062166	49	76	9 331054	600	1080	9 773925	240	240
Cambridge (Amer.)	42 12 02	0 080150	52	97	9 478383	288	733	9 855355	222	222
Dublin obs.	53 12 07	0 060090	48	73	9 304166	670	1155	9 763705	242	242
Edinburg	55 46 02	0 055618	47	67	9 233401	878	1376	9 741011	249	249
Greenwich obs.	51 17 28	0 063466	49	77	9 346396	562	1038	9 780232	238	238
Havannah	23 03 34	0 120000	64	148	9 597658	95	516	10 008045	210	210
Kinderhook	42 11 37	0 080163	52	98	9 478455	289	733	9 855411	222	222
Lancaster	39 51 18	0 084648	54	104	9 501042	249	688	9 874005	219	219
Leon I. obs.	36 16 52	0 091680	55	112	9 529940	202	634	9 902005	216	216
London	51 19 29	0 063406	49	77	9 345714	564	1040	9 779944	238	238
Monticello	37 36 52	0 088372	55	108	9 517220	228	657	9 889004	217	217
Nantucket	41 04 10	0 082308	53	101	9 489723	269	710	9 864379	221	221
Natchez	31 17 36	0 101899	58	125	9 561510	152	577	9 940447	212	212
New Brunswick	40 18 40	0 083766	54	103	9 496889	256	696	9 870397	220	220
New Haven	41 06 35	0 082231	53	101	9 489333	269	711	9 864059	221	221
New York	40 31 19	0 083360	53	102	9 494930	260	700	9 868727	220	220
Newport	41 18 07	0 081863	53	100	9 487457	273	715	9 862529	221	221
Norriton	39 58 37	0 084412	54	103	9 499943	251	691	9 873042	220	220
Oxford obs.	51 34 28	0 062963	50	77	9 340586	576	1054	9 777800	239	239
Paris obs.	48 38 51	0 068207	50	83	9 394413	452	918	9 802627	233	233
Philadelphia	39 45 44	0 084828	53	104	9 501872	248	687	9 874738	219	219
Portland	43 27 32	0 077772	52	95	9 464767	313	761	9 845245	224	224
Richmond obs.	51 16 56	0 063482	49	78	9 346576	562	1038	9 780308	238	238
Rutland	43 24 32	0 077866	52	95	9 465330	312	760	9 845648	224	224
Salem	42 22 04	0 079832	52	98	9 476637	291	731	9 854016	222	222
Washington	38 42 47	0 086870	54	106	9 510949	233	669	9 883002	218	218
Williamsburg	37 04 47	0 090105	55	111	9 524062	211	645	9 895852	217	217

These logarithms are calculated for the obliquity 23° 27′ 40″. The columns marked *lat.* contain the variations of A, B, C, for an increase of 100″ in the reduced latitude. The column *obl.* contains the variations of A, B, C, for an increase of 100″ in the obliquity of the ecliptic. The signs must be changed, if the latitude or obliquity is less than that given in the table.

Example.

Required the values of A, B, C, for Salem, when the obliquity is 23° 27′ 41″·9.

Tabular numbers	0·079832	9·476637	9·854016
Var. for + 1″·9 obliquity	+ 2	— 14	+ 4
Required values	A = 0·079834	B = 9·476623	C = 9·854020

The following rule is adapted to the table of log. sines, &c. numbered xxvii in the third edition of the Practical Navigator, in the margin of which are placed two columns, one marked P. M. containing the double of the time corresponding to the degrees and minutes, allowing 15° for an hour; the other marked A. M. containing the difference between this and 12 hours. *In using tables not having these columns, it will be necessary to turn the time T, mentioned in the rule, into degrees and minutes, and take the log. cotangent of ½T instead of that mentioned in the rule.*

RULE.

Add together the sun's right ascension, the apparent time (counted from noon to noon) and 6 hours, the sum rejecting 24 or 48 hours, if greater than those quantities, is to be called the time T; this is to be sought for in the column of hours of table xxvii, supposing the column A. M. to be increased 12 hours, as in astronomical computation.* The corresponding log. cotangent added to the log. A of

* Thus if the time T is 5 hours, it must be called 5H. P. M. If T is 14

the table gives the log. tangent of the arch G. This log. added to the log. B of the table, rejecting 10 in the index, will be the log. tangent of the arch F; these arches being less than 90° when T is found in the column A. M. otherwise greater. This rule is general except in places situated within the polar circles, which is a case that very rarely occurs. Within the *north polar circle* the supplement of F to 360° is to be taken instead of F; within the *south polar circle*, the supplement of G to 180° is to be taken instead of G; the other terms remaining unaltered. Then the longitude of the nonagesimal is equal to the sum of the arches F, G, and 90°, rejecting as usual 360° when the sum exceeds that quantity.

To the log. C add the log. cosine of the arch G, and the log. secant of the arch F, the sum, rejecting 20 in the index, will be the log. tangent of half the altitude of the nonagesimal.

Example.

Required the altitude and longitude of the Nonagesimal at Salem, September 17, 1811, at 0*h.* 55′ 14″·3, the observed apparent time of the beginning of the eclipse of the sun; the obliquity of the ecliptic being 23° 27′ 41″·9, the reduced latitude 42° 22′ 04″ N, and the sun's right ascension 11*h.* 37′ 33″·9.

11*h.* 37′ 33″·9 ☉'s right ascension.		
0 55 14 ·3 App. time.		
6	A 0 079834	
T 18 32 48 ·2	Cotang 0 062375=log. cotang of ½T turned into deg.	
G 54 13 03	Tan. 0 142209	Cosine 9 766940
90	B 9 476623	C 9 854020
F 22 34 30	Tan. 9 618832	Secant 34621
Sum 166 47 33=Long. Nonages.		24 20 41½ Tan. 9 655581
		48 41 23 Altitude Nonages.

....

hours it must be called 2 H. A. M. the corresponding log. tangent is equal to the log. tangent of ½T turned into degrees and minutes in the usual way.

Demonstration.

Let EQ (Plat. 2, Fig. 2) be the equator, ♑ ♋ the ecliptic, ♈ the first point of Aries, p the north pole of the ecliptic, P the north pole of the equator, Z the zenith of the place of observation, P p the obliquity of the ecliptic, P Z the north polar distance of the point Z. Then as in President Willard's paper, in the first volume of the Memoirs of the Academy, p Z is the altitude of the nonagesimal $=$ H; and the angle ♈ p Z its longitude $=$ L. The angle ♈ P Z is equal to the right ascension of the meridian R, found by adding the apparent time to the sun's right ascension.

In the spherical triangle p P Z put $S = \frac{1}{2}(PZ + Pp)$, $D = \frac{1}{2}(PZ - Pp)$, the angle $pPZ = 90° + R = T$, $G = 180° - \frac{1}{2}(PpZ + pZP)$, $F = 180° - \frac{1}{2}(PpZ - pZP)$. Hence $PpZ = 360° - F - G$ and $90° - PpZ =$ ♈ $pZ =$ longitude of the nonagesimal L becomes $L = 90° + F + G$, rejecting as usual the 360°. Then, by the noted rules of Napier (marked (7) (8) in page 653 of the third edition of the Navigator) we have Cos. S : Cos. D : : Cot. $\frac{1}{2}$T : Tang. (180° — G) and Sine S : Sine D : : Cot. $\frac{1}{2}$T : Tang. (180° — F). Dividing the terms of the last analogy by the corresponding ones of the former, and putting $\frac{\text{Sin. S}}{\text{Cos. S}} = \text{Tang. S}$, $\frac{\text{Sine D}}{\text{Cos. D}} = \text{Tang. D}$, and noting the signs of the terms in the usual way, in order to ascertain the affection of F and G, by putting Tang. (180° — G) $=$ — Tang. G, Tang. (180° — F) $=$ — Tang. F, we have Cos. S : Cos. D : : Cot. $\frac{1}{2}$ T : — Tang. G; and Tang. S : Tang. D : : Tang. G : Tang. F. Or, in putting $\frac{\text{Cos. D}}{\text{Cos. S}} = A$, $\frac{\text{Tang. D}}{\text{Tang. S}} = B$, Tang. G $=$ — A Cot. $\frac{1}{2}$ T, Tang. F $=$ B Tang. G.

The quantities A, B, are evidently the natural numbers corresponding to the logarithms A, B, of the preceding table. The form-

ulas for Tang. F and Tang. G, furnish in logarithms the rule above given for calculating F and G to be substituted in the value of $L = 90^\circ + F + G$.

When R is in the ascending signs, as in Fig. 2, and Z is situated without the polar circles, D, S and $\frac{1}{2}$ T must be acute. In this case the formula for Tang. G and Tang. F become negative, consequently G, F will be obtuse. In Fig. 3, R is in the descending signs, and Z without the polar circles, D and S are acute, $\frac{1}{2}$ T obtuse, and its tangent becomes negative; hence, by the formula tang. G and tang. F are positive, and F, G acute. Consequently G and F are of a different affection from $\frac{1}{2}$ T agreeable to the rule.

If the polar distance P Z (Fig. 2, 3) decrease and become equal to Pp, D, B and F will be $= 0$. By decreasing farther the value of P Z, the point Z will fall within the north polar circle, P Z will be less than Pp, and D, B will become negative, and F change its sign. Hence to make use of the formula $L = 90 + F + G$, it will be necessary, in this case, to write $360^\circ - F$ instead of F. On the contrary, if the polar distance P Z (Fig. 2, 3.) be supposed to increase, D and S will remain acute until $PZ = 180^\circ - Pp$, then $S = 90^\circ$ and its cosine $= 0$, consequently A and tang. G will become infinite and $G = 90^\circ$. Beyond that point, within the south polar circle, S will exceed 90°, its cosine will be negative, A, B, will be negative, and tang. G will change sign, consequently the supplement of its former value must be taken. These agree with the rule and include all the cases.

Again (by § 10 page 653 of the Navigator) we have in the triangle PpZ, Cos. $\frac{1}{2}(PpZ - PZp)$: Cos. $\frac{1}{2}(PpZ + PZp)$: : tan. $\frac{1}{2}(Pp + PZ)$: tang. $\frac{1}{2}pZ$, which in symbols is Cos. $(180^\circ - F)$: Cos. $(180^\circ - G)$: : tan. S : tang. $\frac{1}{2}$ H, or by reduction cos. F : cos. G : : tang. S : tang. $\frac{1}{2}$ H, whence the rule for finding the half altitude of the nonagesimal is easily deduced. It may be observed that this rule

gives the distance from the north pole of the ecliptic to the zenith, which may sometimes be obtuse and equal to the supplement of the actual altitude.

Cor. From the above demonstration it is evident, that the difference of the arches F, G (or its supplement to 360°) is equal to the angle PZp. This angle is useful in finding the correction of the altitude and longitude of the nonagesimal, from an error in the latitude of the place. Thus if the latitude were increased by the quantity Za (in the case of Fig. 3) and the perpendicular ab were let fall on the arch pZ, the altitude of the nonagesimal would be decreased by $Zb = Za \times \cos. PZp$, and as $ab = Za \times \text{sine } PZp$, the longitude of the nonagesimal would be decreased by the angle $Zpa = Za \times \frac{\text{Sine } PZp}{\text{Sine } Zp}$.

Thus in the preceding example the difference of the arches G, F is $31° 38' 33'' = PZp$, its cosine is ·851, its sine ·525. Hence if the increment of latitude Za be $100''$, the decrement of the altitude of the nonagesimal Zb will be $85''{\cdot}1$, and the arch $ab = 52''{\cdot}5$, which divided by the sine of the altitude pZ ·7511 will give the decrement of the longitude of the nonagesimal $69''{\cdot}9$.